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A MODEL FOR PREPARATIVE SCALE GAS CHROMATOGRAPHY

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SUMMARY

A model for preparative scale gas chromatography is derived, first by choosing only a few of the many phenomena involved in such a process, then by using a new description of the process itself as well as a better mathematical method to account for it. The new description of the phenomena makes use of discontinuities of the solute mole fraction which implies that discontinuous as well as continuous equations can be written. Use of the method of characteristics for the continuous equations allows the two types of solution to be combined into a very general description of the movement of solutes down a chromatographic column. Examples of application are given for the cases of one and two solutes.

INTRODUCTION

Much theoretical work has been published about chromatography. Most of the literature dealt with phenomena involved in a chromatographic process whereas a few theories gave a complete model for such a process. Only the latter will be considered here, and a short review of them is given in Table I. As shown by Table I, the theories of gas chromatography (GC) appeared later because GC itself is a relatively young technique. Before presenting a new model for GC at finite concentration, it is necessary to survey the previous ones so that the essential features of the model can be pointed out.

The first theories of chromatography, which were derived for liquid chromato-

TABLE I

THE VARIOUS GENERAL THEORIES OF CHROMATOGRAPHY

Type	*Year*	*Author*	*Main reference*
Liquid; finite concn.	1940s	WILSON; DEVAULT	1, 2
Ion exchange; finite concn.	1940s	GLUECKAUF	3
Gas; analytical	1950s	GIDDINGS	4
Gas; finite concn.	1955	BOSANQUET AND MORGAN	5
Liquid; finite concn.	1960s	RACHINSKII	6
Ion exchange; finite concn.	1970	KLEIN AND HELFFERICH; TONDEUR	7, 8
Gas; finite concn.	1970	GUIOCHON AND JACOB	9

graphy, are based on partial differential equations[1,2]. These theories, which presented a complete set of hypotheses allowing the basic equations to be derived, accounted for the finite concentration and partly for the occurrence of very steep slopes on one side of a peak[2]. Further steps towards the greater use of mathematics were made in the case of liquid chromatography by RACHINSKII[6], who introduced better mathematical methods. However, GC was discovered, and made use of much smaller amounts of solute so that it was soon used under analytical conditions at almost infinite dilution. It is therefore not surprising that the models devised at that time were statistical[4] and no longer used partial derivatives. However, the theories devised by GIDDINGS[4] in the 1950s were still based on liquid dynamics. Immediately afterwards, attempts to deal with finite concentrations in GC were made for example by BOSANQUET AND MORGAN[5]. However, they were very limited in scope. In fact, the methods used in these models were all very similar to those used by DEVAULT[2], and all the papers in the literature referred to his work. However, if further steps were to be taken, it was obvious that something had to be added. This could be done using more suitable mathematical methods for both liquid and gas chromatography. It could also mean adding new phenomena so that the models would be closer to reality. These features can be found in the two latest models to be published, that of TONDEUR[8] and that of GUIOCHON AND JACOB[9]. This last model is presented here, first the phenomena, then the mathematical method, and finally the application of the model.

THE MAIN PHENOMENA

When one speaks of preparative scale chromatography, one does not always indicate exactly which model has been chosen or refer to it correctly. A clear basis will be given here for the model presented, so that an easy comparison can be made with others. The main phenomena involved are shown in Table II. The first group of phenomena can be neglected for various reasons[9] of which the essential ones are the following.

TABLE II

THE PHENOMENA THAT INTERVENE IN FINITE CONCENTRATION GAS CHROMATOGRAPHY

Included in model	*Excluded from model*
Isotherm	Local pressure Temperature
Molar volume variation	Diffusion Flow
Discontinuities	Column

The local pressure influences the flow of the mobile phase, but if this factor was taken into account, the resulting equations would be far too complicated. It is indeed a very important phenomenon and should actually be dealt with at finite concentration to be able to obtain physical data for the solutes. However, our present purpose is to achieve a fairly general model and if the pressure is included this aim will no longer be possible, whereas results in good agreement with the experimental ones are obtained with models having this feature. Besides, if the theories of GC are considered in detail, one realizes that very few of them fully include the effect of pressure. For example, in the statistical theory of Giddings[4], the influence of decompression on the mobile phase velocity is not included although it modifies the statistical distribution of the solute.

Temperature and diffusion can be considered as minor phenomena as far as finite concentrations are concerned, as will be seen later. Finally, only a piston-flow or a mono-dimensional column will be considered.

Thus there remain only the three basic characteristic features of our model of chromatography at high concentration: the solubility isotherm, the differences in the molar volumes of the solute, and the occurrence of concentration discontinuities.

The solubility isotherm relates the concentration between the two phases of a solute at equilibrium between the mobile and the stationary phases. When the mole fractions become important, the law relating the number of solute molecules in the mobile phase to that in the stationary phase is no longer linear but can have almost any form.

The variation in molar volumes illustrates the fact that the partial molar volumes of the molecules of solute in the gas and liquid phases are very different. The molar volume in the liquid phase can in fact be neglected compared with that in the gas phase. It follows, therefore, that when a molecule is sorbed, it creates a void in the gas phase and modifies the pressure profile. In the regions where molecules are mostly sorbed, that is at the front of a peak, the pressure will tend to decrease, and conversely it will increase at the back of a peak where the molecules desorb. This generates a greater pressure gradient within the peak than exists outside it and, according to Darcy's law, the mobile phase velocity will also be greater within the peak.

There remains now to be described the most important phenomenon, the occurrence of discontinuities. Discontinuities are a mathematical concept and they may seem far removed from chemistry. A quick look to the asymmetry of peaks will

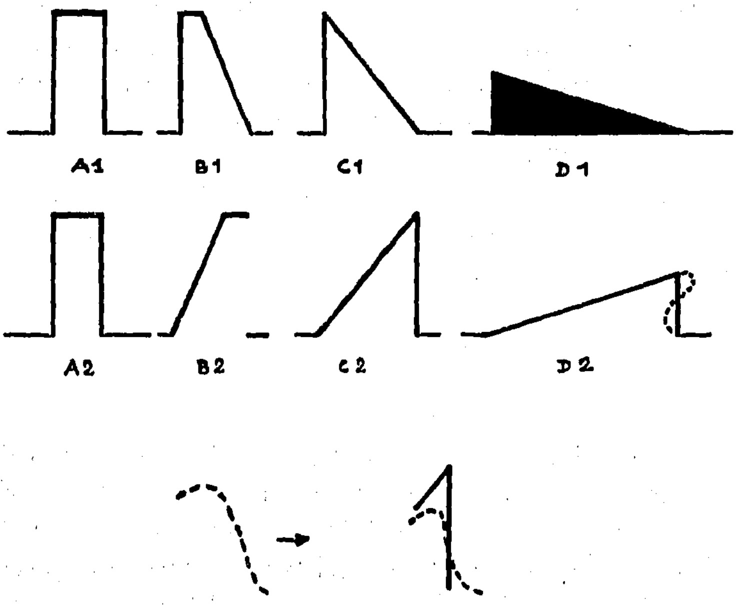

Fig. 1. Introduction to discontinuities and shocks. B1, C1 and D1 are the chromatograms of the injection A1 at various increasing column lengths; B2, C2, D2 and A2 are similar. The asymmetry effects are opposite in the two lines; C3 shows the effect of diffusion on peak C2.

clarify the position. First, what is a chromatogram? From the results presented by recorders, the common picture of a chromatogram is similar to those on Fig. 1, *i.e.* a line, almost a straight one, with peaks on it. However, a correct representation would be to blacken the inner area of a peak, for a recorder really records the distance from the base line (Fig. 1, D1). Peaks A1 and A2 are injection peaks. These peaks are eluted in the sequence A, B, C, D and show increased asymmetry of opposite sign on the two lines. This is only a schematic view but a very important question arises: what do the vertical lines mean, and do they have any physical sense? In fact, vertical lines are not realistic since no recorder can have a zero response time. Besides, diffusion intervenes and, for example, C2 will be turned into C3 owing to diffusion, pictured for a Gaussian peak in A3. However, if these two phenomena, both of which tend to lower the concentration gradient, are neglected, then the usual profiles such as C3 become similar to C2 with a vertical line. However, such lines may not even exist. They are related only to the recorder. They are the unescapable trace of the movement of the recorder pen from a stable position on the top of the peak to the base line, another stable position. The solute concentration in the gas phase changes abruptly from the value corresponding to the top of the peak, to zero, without taking any of the intermediate values. There should not be anything between the top and the base line, as in B2, and this is just what the mathematicians call a discontinuity. If use is made of the suggestion that the inner area of a peak should be blackened, then the vertical lines should be the border of the blackened areas, but they do not belong to them.

Mathematics will not be considered here and applications will be discussed. It is shown in the next paragraph that if the discontinuities are dealt with using the same mathematical methods as for the rest of the peak, pictures such as D2 are obtained. Obviously this cannot be accepted, although it does not seem unnatural to the eye. In fact, if the blackened area is used, it can be seen that various areas will be blackened two or three times. The only solution found to this until now was to draw a new discontinuity that would ensure the mass conservation of the folded areas[2]. This is an ingenious way of solving the problem but it does not follow the principles underlying the method of blackening the areas and is quite arbitrary. It seems better to admit from the beginning the existence of such discontinuities and to admit that they can have physical and dynamic properties. This concept is fairly new in chemistry, but in everyday life nobody doubts any more that sound waves and shocks exist. All the vertical lines in Fig. 1 pertain to the same species as the supersonic "bangs". It will now be shown how these shocks are included in the theory, and for this purpose some mathematics must be used.

MATHEMATICAL METHODS

Until now, no mathematical method has been shown to be capable of dealing with the discontinuities as well as with the continuous part of the concentration profile. This continuous part can be dealt with using the usual functions, which is not the case for discontinuities. The interaction of discontinuities and the continuous part must also be studied.

The discontinuities have already been studied extensively in spite of their fairly complicated nature. This stems from the fact that a very simple method of creating a stable discontinuity, that is a shock, is to achieve frontal analysis (except in the case of some solubility isotherms). The reason will become clear in the next paragraph. Thus the theory is so simple and so well known that it needs not be repeated here.

The continuous part, on the contrary, although fairly easy to define, requires somewhat more sophisticated methods to deal with. The equations for the propagation of a single solute by the carrier gas are:

$$(1 + k')\frac{\partial X}{\partial t} + \frac{\partial}{\partial z}(Xu) = 0$$

$$\frac{\partial(1 - X)}{\partial t} + \frac{\partial}{\partial z}((1 - X)u) = 0 ,$$

where X is the mole fraction, u the mobile phase velocity, k' the capacity ratio, t the time and z the abscissa along the column.

These are mere mass conservation equations, one for the solute and one for the carrier gas, provided that we use the model defined in the previous sections[9].

The two sets of equations are partial differential equations and together they build a system. Each of these equations is quasi-linear, as only first-order partial derivatives appear and as their factors are functions of the functions themselves. The system itself is hyperbolic. These properties allow a solution of the problem to be sought by using the method of characteristics. In fact, this result is quite general and is valid for any number of solutes.

Characteristics are lines plotted in a two-dimensional space, built on time and abscissa in the column as coordinates. Therefore characteristics can be considered as trajectories. On these lines, the partial differential equations turn into simple differential equations, allowing either simpler calculation or geometrical construction. It can be shown[9] that there are as many families of characteristics as there are solutes, plus one. This "supplementary" one is related to the carrier gas and in fact happens to be the same in all systems. This last family is called ϱ_1, and correlates the various mobile phase velocities along the column at a given moment.

The equations of the characteristics of the families 1 and 2 for a pure compound are:

$$\varrho_1 \begin{cases} dt = 0 \\ \dfrac{\partial}{\partial z}(u(1 + k'(1 - X))) = 0 \end{cases}$$

$$\varrho_1, \varrho_2 \in \mathbb{R}^2(z, t)$$

$$\varrho_2 \begin{cases} \dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{u}{1 + k'(1 - X)} \\ \mathrm{d}X = 0 \end{cases}$$

The only restriction to the use of characteristics is that two characteristics of the same family may not intersect. When this occurs the solution can no longer be found by this method. Then the physical model indicates that a new physical phenomenon intervenes, *i.e.* a shock, as will be discussed later.

There remains to be discussed the interaction of shocks or discontinuities with the continuous part of the concentration profiles. This interaction is necessarily derived from the definition given earlier: on the one hand, the discontinuity is defined as the border of the continuous part, and on the other hand, it has its own laws of propagation. The equations show that the expressions of these two properties are not identical; therefore some interaction must occur between the discontinuity and the neighbouring continuous part. One way of expressing this is to assume that a discontinuity is defined by its neighbouring characteristics, *i.e.* those corresponding to the two values of the concentration which determine this discontinuity, but that the discontinuity is propagated according to its own laws. Then, if characteristics do not intersect the path of a discontinuity, this discontinuity cannot exist and it falls into a continuous part. If characteristics do cut the path of the discontinuity, the discontinuity is stable and is a shock[9].

It will now be shown how the theory can account for the phenomena observed in preparative scale GC.

APPLICATIONS OF THE THEORY

The first problem concerns the origin of discontinuities. It has been shown that a discontinuity could be stable if the characteristics intersect its path, but this implies that the discontinuity already existed, as in Fig. 1. However, discontinuities can appear in the middle of a continuous part in the following way. It has been said that characteristics of one family might not interfere one with the other. However, the velocity of a characteristic depends on the mole fraction attached to this line. Therefore, neighbouring characteristics of a given family can either diverge or converge but are very seldom parallel. When they converge, they will tend to cut each other. When this happens, the continuous solution is no longer valid and a discontinuity is then created. The mathematics show that a characteristic is similar to a zero discontinuity and therefore the interaction of two characteristics can be dealt with in the same way as for that of a characteristic and a discontinuity; a discontinuity created in that way will tend to be stable and to grow.

Other applications can be made from the fact that the slopes of characteristics depend on the mole fraction, and a typical example is the description of the elution of a single peak and of the build-up of its asymmetry. This is plotted in Fig. 2, where it is assumed that the slope of the characteristics increases with increasing mole fraction. Also, it can be derived mathematically, and was shown in the two sets of equations, that for a single compound the mole fraction is constant on any given characteristic. The original injection peak is O. It is rectangular and has two discontinuities, OO′ and O″O‴. The second discontinuity, OO′, is not stable and collapses into a set of characteristics, so only O″O‴ is a shock. To obtain the elution of the peak, the path of the shock and the characteristics must be drawn. Between O and B, the characteristics issued from O′O″ intersect the shock, and, all corresponding to the same concentration, allow the shock to remain of constant height. What occurs between O and B is exactly what occurs in frontal analysis and explains how the initial discontinuity can be stable. However, after B this is no longer valid and the characteristics intersecting the shock have a lower mole fraction. It follows that the height of the shock decreases as well as its velocity, as can be shown from the equations[9].

This is in fact natural, as, owing to the characteristic OABC (dotted line), the peak broadens and therefore, if mass is to be conserved, the peak height must decrease.

The chromatograms in O, A, B and C are the same as chromatograms A1, B1, C1

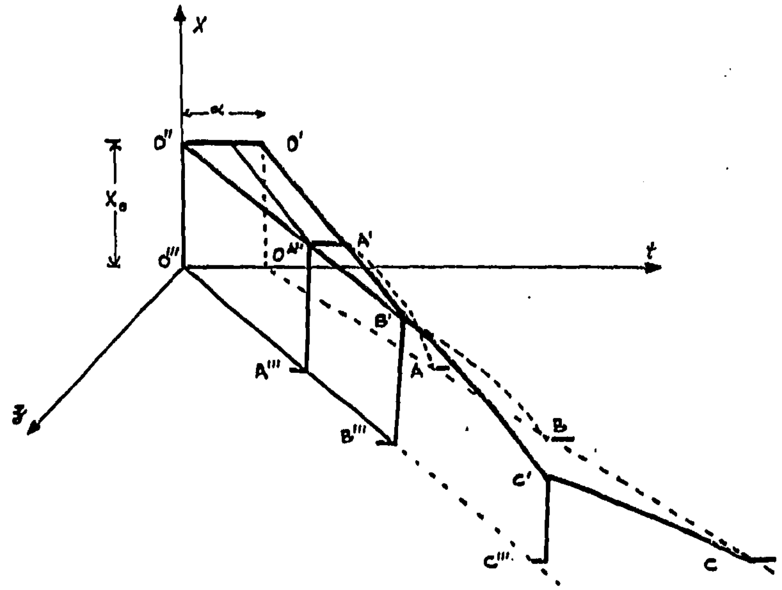

Fig. 2. Elution of a single peak. The injection peak is in O. There it turns into A then B and C. X is the mole fraction, z the abscissa, t the time.

and D1 in Fig. 1 and this shows how characteristics account for the build-up of the peak asymmetry.

If some physical measurements are to be made, it is best to measure data that do not change during elution. This means that it will be easier to measure data represented by straight lines in Fig. 2. However, there are few straight lines there that one can rely on. The path of the shock cannot be used. The characteristics could be used, but as their slopes depend on the mole fraction, one must be sure of which characteristic is being studied.

In the end, the most reliable line is OABC, that is, the characteristic of zero mole fraction. This is exactly what KEULEMANS[10] discovered experimentally when he defined the initial and final retention volumes, based on this characteristic for the two possible maximum opposite asymmetries and not on the peak top.

Other features can be derived, for example for the separation of two solutes. Separation of one solute from a mixture means, in GC, that on the chromatogram a zone can be found where only this solute is mixed with the carrier gas. Such a zone can be found even though the separation is not completed, in which case it will be located next to a zone where the two solutes are mixed with the carrier gas. The concentrations of the solutes can vary continuously or discontinuously from one zone to the other, across the boundary. The theory of finite concentration GC indicates[9] that the paths of these boundaries are boundary lines that are shocks if the boundary is discontinuous or a characteristic if it is continuous. Furthermore, it indicates that these boundary lines are issued from the limits of the boundary condition at the column inlet, that is, the limits of the injection plug, and that there are as many lines issued as there are solutes. An example of such a process is shown in Fig. 3 for two solutes. The injection lasts from S1 to S2 in the (z,t) plane at the inlet of the column $(z=0)$. The boundary lines issued from S1 are C3 and C4, those from S2 are C1 and C2. As was just defined, these four lines define five domains, $\mathscr{D}_i$, in each of which the number of solutes is constant. In $\mathscr{D}_3$ lies the mixture of the two components and

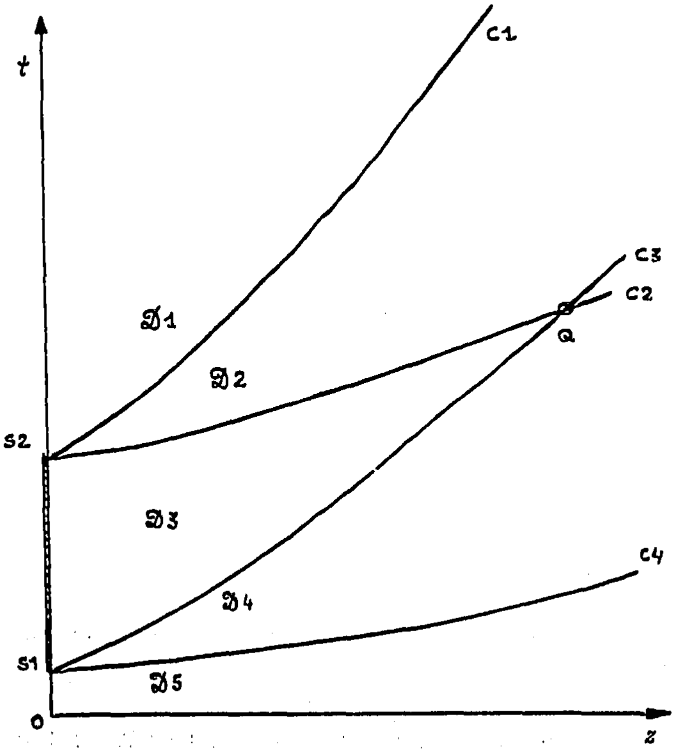

Fig. 3. Separation of a mixture of two compounds. *t* is the time, *z* the abscissa, C_i are boundaries (either shocks or characteristics) i are the various domains accounting for the separation. Q marks the end of the separation process.

carrier gas, in $\mathscr{D}_1$ and $\mathscr{D}_5$ only carrier gas can be found, and in $\mathscr{D}_2$ and $\mathscr{D}_4$ the slower and faster compounds occur, respectively. $\mathscr{D}_3$ is bound by C2 and C3, and therefore when these lines intersect no further mixing can take place. Point Q thus marks the completion of the separation process.

Naturally, with the use of computers, one can obtain much more data on this process.

CONCLUSION

The above description is only a summary of the theory and application of the model. However, it includes the two main points of it: a new mathematical method (characteristics) and a new concept (shocks). It is not argued that we were the first to think of these, but we feel that we are the first to implement them, at any rate for GC.

The characteristics are essential as they combine all the previous trials for the description of the propagation of continuous concentration profiles at finite concentration. The shocks are essential in that without them no correct description of the elution of a peak can be achieved in the general case. Also, they allow frontal and elution chromatography to be combined. It seems that at present much progress is being made using the two concepts and it is hoped that they will allow a significant step in the understanding of chromatography at finite concentration.

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